

# APPLICATION OF IMMERSION AND INVARIANCE TECHNIQUE FOR IMPROVING SMIB AND TMIB PERFORMANCE

Siva Rama Krishna Paladi<sup>1</sup> and C. Bhavani Manogna<sup>2</sup>

<sup>1</sup>Gitam university Hyderabad

<sup>2</sup>M.Tech Scholar Gitam University Hyderabad

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**Abstract**—The problem of transient stabilization of electrical power systems has been an active area of research in recent years. In this paper we address the performance of different control laws for stabilization of the Single Machine Infinite Bus (SMIB) system and a Two Machine Infinite Bus (TMIB) system with Controllable Series Capacitors (CSCs) as actuators. The SMIB and a TMIB systems are described by the swing equation model and the CSCs are modeled by the injection model. The control laws are derived using nonlinear control techniques- Immersion and Invariance (I&I).

**Index Terms:** SMIB system, TMIBsystem, CSC, I&I.

## 1. INTRODUCTION

Traditional analysis and control techniques for power systems are undergoing a major reassessment in recent years. See [1] for an account on the new issues in power system operations. Power systems exhibit various modes of oscillation due to interactions among system components. Many of the oscillations are due to synchronous generator rotors swinging relative to each other. Transient stability is concerned with a power systems ability to reach an acceptable steady-state following a fault. Conventionally, linear controllers are used to improve the transient performance. Given the highly nonlinear nature of the power system models the applicability of linear controller design techniques for transient stability enhancement is severely restricted. On the other hand, the application of more promising nonlinear control methods has attracted much attention in the literature to replace the traditional Automatic Voltage Regulator (AVR) and the Power System Stabilizer (PSS) control structure. In [2], nonlinear control using turbine control, and excitation control has been proposed. In [3]–[6], feedback linearization is applied to the control of a single machine as well as multi-machine systems, using output feedback and state observers. However, robustness problems, both against parameter uncertainties and unmodeled dynamics, of these nonlinearity cancellation schemes remains unanswered. In [7], nonlinear controller design of thyristor controlled series compensation is presented for damping inter-area power oscillations.

An important factor, which decides the capacity of a transmission line to transfer the electrical power across the network, the stability margin of the power system, is the reactance of the transmission line. The concept of Flexible AC Transmission System (FACTS) relies on the use of such power electronic devices, and offers greater control of power flow, secure loading and damping of power system oscillations see, e.g., [8]. The devices can be classified into those operating in shunt with the power line such as Static Var Compensator (SVC), in which cases the injected currents are controlled and those operating in series with the power line such as Controllable Series Capacitor (CSC), Unified Power Flow Controller (UPFC), in which cases the inserted voltages are controlled. Recently the application of nonlinear control strategies with controllable series devices has been investigated for improving the transient stability of a power system [7]–[9]. In [10], the IDA-PBC strategy is used for transient stabilization of a synchronous generator using a CSC. Further in [11], the I&I strategy has been used to stabilize the nonlinear swing equation model of the SMIB using a CSC.

In this paper we address the problem of transient stabilization of the SMIB and TMIB systems using CSCs. The power systems are modeled using the swing equation model and the CSCs are modeled by the injection model [12]. The paper is organized as follows: In Section 2 we briefly introduce the control synthesis methodologies. In Section 3 we describe the SMIB system. The control laws are derived using I&I control technique, and comparison study using simulation plots. In Section 4 we describe the TMIB system. The control laws are derived and comparison study is performed using simulation plots. And finally Section 5 concludes the paper.

## 2. CONTROL STRATEGIES

In this section we briefly describe the control technique for transient stabilization of SMIB and TMIB. We consider nonlinear control techniques- Immersion and Invariance (I&I).

## 2.1 Immersion and Invariance

Immersion and invariance[15] relies upon the notions of system immersion and manifold invariance. The basic idea of I&I is based on (a) immersing a lower order desired target dynamics onto a manifold in the original space, and (b) matching the closed-loop system with the immersed system asymptotically. The control objective is to make the immersed manifold attractive and invariant. The control objective is to make the immersed manifold attractive and invariant. The main result of [1] is now stated .

Theorem:[15] Consider the state space model of the system

$$\dot{x} = f(x) + g(x)u \quad (1)$$

where  $f(x)$  and  $g(x)$  are smooth functions ,with state  $x \in \mathbb{R}^n$  and control  $u \in \mathbb{R}^m$ , with an equilibrium point,  $x_* \in \mathbb{R}^n$  to be stabilized . Let  $p < n$  and assume we can find mappings

$$\alpha: \mathbb{R}^p \rightarrow \mathbb{R}^n, \pi: \mathbb{R}^p \rightarrow \mathbb{R}^n, c: \mathbb{R}^p \rightarrow \mathbb{R}^m,$$

$$\phi: \mathbb{R}^n \rightarrow \mathbb{R}^p, \Psi: \mathbb{R}^n \times \mathbb{R}^{n-p} \rightarrow \mathbb{R}^m$$

such that the following hold.

1) (H1) (Target system ) The system

$$\dot{\xi} = \alpha(\xi) \quad (2)$$

with state  $\xi \in \mathbb{R}^p$  has asymptotically stable equilibrium at  $\xi_* \in \mathbb{R}^p$  and  $x_* = \pi(\xi_*)$ .

2) (H2) (Immersion condition) For all  $\xi \in \mathbb{R}^p$

$$f(\pi(\xi)) + g(\pi(\xi))c(\pi(\xi)) = \frac{\partial \pi}{\partial \xi} \alpha(\xi). \quad (3)$$

3) (H3) (Implicit Manifold) The following set identity

$$\begin{aligned} \{X \in \mathbb{R}^n \mid \phi(x) = 0\} \\ = \{X \in \mathbb{R}^n \mid x = \pi(\xi) \text{ for some } \xi \in \mathbb{R}^p\} \end{aligned} \quad (4)$$

4) (H4) (Manifold attractive and trajectory boundedness all

Trajectories of the system

$$\dot{z} = \frac{\partial \phi}{\partial x} [f(x) + g(x)\psi(x,z)] \quad (5)$$

$$\dot{x} = f(x) + g(x)\psi(x,z) \quad (6)$$

are bounded and satisfy

$$\lim_{t \rightarrow \infty} z(t) = 0 \quad (7)$$

Where  $z = \phi(x)$  and  $u = \psi(x)$ .

Then  $x_*$  is an asymptotically stable equilibrium of the closed loop system

$$\dot{x} = f(x) + g\psi(x, \phi(x)).$$

The result in theorem implies that the stabilization problem for the system(1) can be divided into two sub problems .First, given the target dynamical system  $\dot{\xi} = \alpha(\xi)$  which is locally asymptotically stable and of dimension strictly smaller than the dimension of  $x$ , find if possible ,a manifold  $M$  described implicitly by  $\{x \in \mathbb{R}^n \mid \phi(x) = 0\}$  and in parameterized form by  $\{x \in \mathbb{R}^n \mid x = \pi(\xi), \text{ for some } \xi \in \mathbb{R}^p\}$ , which can be rendered invariant and such that the restriction of the closed-loop system to  $M$  is described by the target dynamics. The mapping  $\pi: \xi \rightarrow x$  is an immersion, that is, the rank of  $\pi$  is equal to the dimension of  $\xi$ . Secondly design a control law  $u = \psi(x, z)$  that drives to zero the off-manifold coordinate  $z = \phi(x)$  and keeps the system trajectories bounded.

## 3. I&I Based Controller Synthesis For Stabilization Of SMIB System

In this section we initially present the Modelling of SMIB system and control objective.

### 3.1. Modelling and Problem Formulation

Consider the SMIB system with a CSC as shown in Figure 1. It consists of a synchronous generator connected to the infinite bus or reference bus. The magnitude of the voltage and the frequency for the infinite bus are assumed to be constant. In Figure 1 the generator bus is numbered as 1 and the infinite bus as 2. They are connected to each other through a series combination of the line reactance  $X_{l2}$  and a CSC which is denoted by a reactance  $-jX_c$ . We use the following.

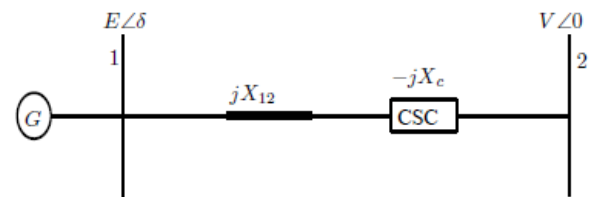


Fig. 1: The SMIB system with a CSC

notation:  $\delta$  is the rotor angle and  $\omega$  is the rotor angular speed deviation with respect to a synchronously rotating reference for the generator. Let  $D > 0$ ,  $M > 0$  and  $P > 0$  be the damping constant, moment of inertia constant and the mechanical power input to the generator, respectively. The dynamics of the synchronous generator is described by the swing equation model as,

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} x_2 \\ \frac{1}{M}[P - Dx_2 - b_1 \sin x_1] \end{pmatrix} + \begin{pmatrix} 0 \\ -\frac{b_1}{M} \sin x_1 \end{pmatrix} u \quad (8)$$

where  $x_1 = \delta$  and  $x_2 = \omega$  are the state variables,  $u$  is the input to the CSC,  $x_{1*}$  is the open loop reactance between buses 1 and 2, and  $b_1 = \frac{EV}{X_{L*}}$ . We assume that the domain of operation as follows

$$D = \{(x_1, x_2) \in S^1 \times \mathbb{R}^1 \mid d_1 < x_1 < \frac{\pi}{2} - d_1, d_1 > 0\}.$$

### 3.2 Control Objective

The open loop operating equilibrium point for the system (12) is given by  $x_* = (x_{1*}, 0)$ . We assume that  $x_*$  is known to us and synthesize a control law  $u$  in order to make the system (12) asymptotically stable at  $x_*$ .

### 3.3 Controller Synthesis:

We use the I&I methodology to synthesize the controller for the SMIB system with a CSC.

### 3.4 Controller Synthesis Using (I&I) Technique

As a first step in the controller synthesis we define a one dimensional target dynamical system as follows

$$\dot{\xi} = -a\xi, a > 0 \quad (9)$$

With the stable equilibrium point  $\xi_*$ , where  $\xi = \xi - \xi_*$

Once we define the target dynamics, we define a mapping

$$\pi: S^1 \rightarrow S^1 \times \mathbb{R}^1 \text{ as follows}$$

$$\pi(\xi) = \begin{pmatrix} \xi \\ \pi_2(\xi) \end{pmatrix}$$

where  $\Pi_2(\xi)$  is to be chosen. Then, with this choice of  $\Pi(\xi)$  and the target dynamics (16), we get the control law as follows

$$u = (P - Dx_2 - b_1 \sin x_1) + aMx_2 + M\gamma(x_2 + a\tilde{x}_1)/b_1 \sin x_1 \quad (10)$$

where  $a$  and  $\gamma$  are the tuning parameters. Note that, as  $x_1$  approaches to zero, the magnitude of the control law grows unbounded.

### 3.5 Simulation Results

We assume the following simulation parameters for the SMIB system as shown figure  $M = \frac{8}{100\pi}$ ,  $D = \frac{0.4}{100\pi}$ ,  $E = V = 1$  (pu),  $b_1 = 2.5$  (pu),  $-\frac{1}{3} \leq u \leq 1$ . To assess the performance of the proposed control laws we assume that a

short circuit fault at the far end of the transmission line at the time  $t = 1$  s for a duration of 0.1 s

We use the following system parameters for the lightly loaded condition: The operating equilibrium point is  $x_* = (0.4556, 0)$  and  $P = 1.1$  (pu). The values of the tuning parameters are chosen as  $a = 5$  and  $\gamma = 7$  for the I&I control law. From the simulations, we can observe the following: The open loop system exhibits heavy and sustained oscillations in  $x_1$  and  $x_2$  as shown by dotted lines in Figure 2. The closed-loop system oscillations decay at a faster rate and settle quickly.

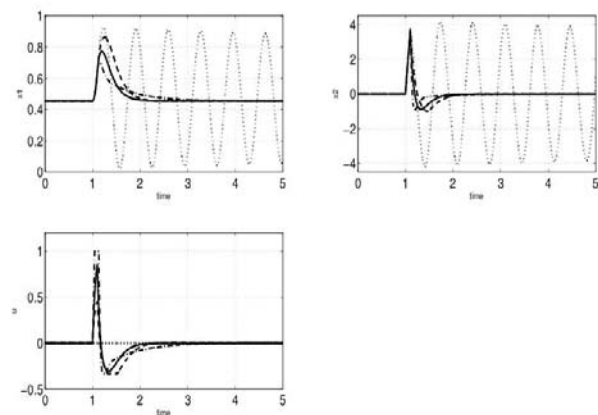


Fig. 3: Response of the SMIB system with a CSC: Solid

line (I& I control law  $a=5, \gamma=7$ ), dashed line (closed loop response  $a=2, \gamma=4$ ), dotted line (open loop response).

## 3. I&I BASED CONTROLLER SYNTHESIS FOR STABILIZATION OF TMIB SYSTEM

In this section we initially present the Modelling of TMIB system and control objective. Next, we derive stabilizing control laws for the TMIB system using power system stabilizer, feedback linearization and I&I techniques. Performance of these control laws are compared, in the case of machine operating at lightly loaded and heavily loaded conditions

### 4.1 Modelling and Problem Formulation:

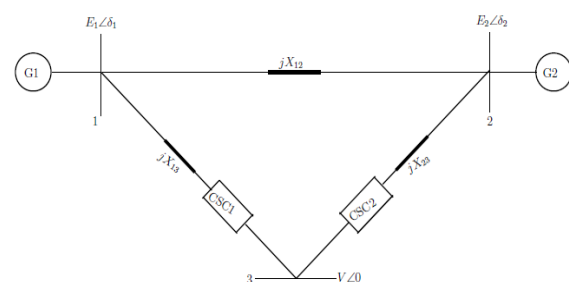


Fig. 4: A Two Machine Infinite Bus System With 2 CSC's

Consider the TMIB system with two CSCs as shown in Figure 4. It shows two generators G1 and G2 connected with the infinite bus. The generator buses for G1 and G2 are denoted by 1 and 2, respectively, and the infinite bus is denoted by 3. We can write the dynamics of the  $i$ -th generator (for  $i=1,2$ )

having  $\delta_i$  as rotor angles and  $\omega_i$  as rotor angular speed deviations as follows

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{pmatrix} = \begin{pmatrix} x_3 \\ x_4 \\ \frac{1}{M_1} [P_1 - b_{12} \sin(x_1 - x_2) - b_1 \sin x_1 - D_1 x_3] \\ \frac{1}{M_2} [P_2 - b_{12} \sin(x_2 - x_1) - b_2 \sin x_2 - D_2 x_4] \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ -\frac{b_1}{M_1} \sin x_1 & 0 \\ 0 & -\frac{b_2}{M_2} \sin x_2 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \quad (11)$$

where  $x_1 = \delta_1, x_2 = \delta_2, x_3 = \omega_1$ , and  $x_4 = \omega_2$  are the state variables  $u_1$  and  $u_2$  are the inputs to the CSC and  $D_i > 0, M_i > 0, b_i > 0$  and  $b_{12} > 0$  are the system constants for  $i=1,2$ . We assume the domain of operation as follows

$$D = \{(x_1, x_2, x_3, x_4) \in S^1 \times S^1 \times \mathbb{R}^2 \mid$$

$$d_i < x_i < \frac{\pi}{2} - d_i, d_i > 0, i = 1, 2\}$$

## 4.2 Control Objective

The open loop operating equilibrium point for the system (18) is given by  $x_* = (x_{1*}, x_{2*}, 0, 0)$ . We assume that  $x_*$  is known to us and synthesize a control laws  $u_1$  and  $u_2$  in order to make the system (18) asymptotically stable at  $x_*$ .

## 4.3 Controller Synthesis using I&I technique

We define the target dynamical system as follows

$$\dot{\xi}_1 = -a_1 \tilde{\xi}_1, a_1 > 0 \quad (12)$$

$$\dot{\xi}_2 = -a_2 \tilde{\xi}_2, a_2 > 0 \quad (13)$$

with stable equilibrium point  $\xi_* = (\xi_{1*}, \xi_{2*})$ , where  $\tilde{\xi}_* = (\tilde{\xi}_{1*}, \tilde{\xi}_{2*})$  for  $i=1,2$ .

Once we define the target dynamics, we define a mapping  $\Pi: S^1 \times S^1 \rightarrow S^1 \times S^1 \times \mathbb{R}^2$  as follows

$$\Pi(\xi) = \begin{pmatrix} \xi_1 \\ \xi_2 \\ \Pi_3(\xi) \\ \Pi_4(\xi) \end{pmatrix}$$

where  $\Pi_3(\xi)$  and  $\Pi_4(\xi)$  are to be chosen. Then, with this choice of  $\Pi(\xi)$  and the target dynamics (25) and (26), we get the control laws as follows

$$u_1 = \frac{P_1 - b_{12} \sin(x_1 - x_2) - b_1 \sin x_1 - D_1 x_3 + a_1 M_1 x_3 + M_1 \gamma_1 (x_3 + a_1 \tilde{x}_1)}{b_1 \sin x_1} \quad (14)$$

$$u_2 = \frac{P_2 - b_{12} \sin(x_1 - x_2) - b_2 \sin x_2 - D_2 x_4 + a_2 M_2 x_4 + M_2 \gamma_2 (x_4 + a_2 \tilde{x}_2)}{b_2 \sin x_2} \quad (15)$$

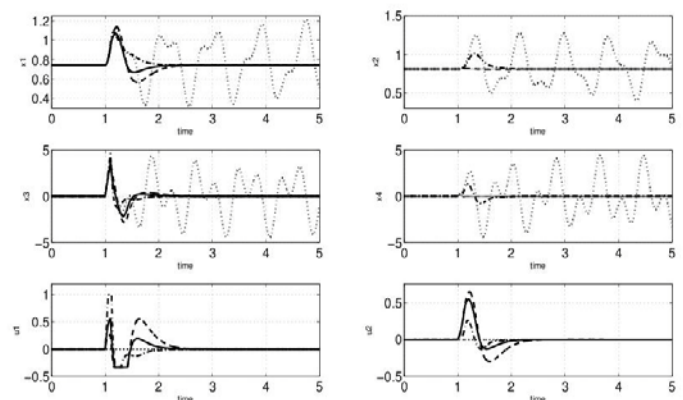
Where  $a_1, a_2, \gamma_1$ , and  $\gamma_2$  are the tuning parameters. Note that as  $x_1$  and  $x_2$  approaches to zero, the magnitude of the control laws grows unbounded.

## 4.4 Simulation results

We use the following system parameters for the TMIB

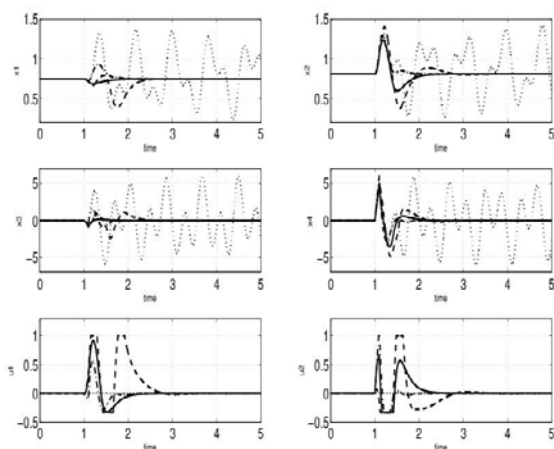
system shown in figure(4):  $M_1 = M_2 = \frac{8}{100\pi}, D_1 = D_2 = \frac{0.4}{100\pi}, b_1 = b_2 = 2(\text{pu}), v=1(\text{pu}), \frac{1}{3} \leq u_1 \leq 1, \frac{1}{3} \leq u_2 \leq 1$ .

To assess the performance of the proposed control laws we assume the following three transient condition (a) Fault at generator G1 (b) Fault at generator G2 and (c) Simultaneous faults occurs at the generators G1 and G2, at time  $t = 1$  s for a duration of 0.1 s.

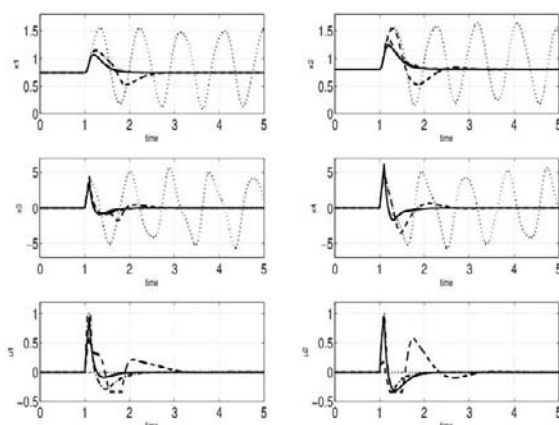


**Fig4.1 Response of TMIB system-with I&I control law and the fault at generator G1. Dashed line represents (closed loop response  $a_1=a_2=\gamma_1=\gamma_2=3$ ), solid line represents (I&I control law  $a_1=8, a_2=4, \gamma_1=10, \gamma_2=8$ ), Dotted line represents (open loop response)**

In all the three cases mentioned above, the open loop system exhibits heavy and sustained oscillations in  $x_1$ ,  $x_2$ ,  $x_3$  and  $x_4$  as shown by dotted lines in Figures. The closed-loop response is plotted for two different sets of tuning parameters, and it shows significant improvement in terms of the peak overshoot as well as the settling time. Further, we observe that an increase in the values of the tuning parameters results in, decrease in the peak overshoot and the oscillations die out quickly in all three transient conditions. The third fault condition (simultaneous faults) is quite severe, however the proposed control law stabilizes the closed-loop system efficiently and swiftly.



**Fig. 4.2: Response of TMIB system-with I&I control law and the fault at generator G2. Dashed line represents(closed loop response  $a_1=a_2=\gamma_1=\gamma_2=3$ ),solid line represents(I&I control law  $a_1=8,a_2=4,\gamma_1=10,\gamma_2=8$ ) Dotted line represents (open loop response)**



**Fig. 4.3: Response of TMIB system-with I&I control law and the fault at generator G1 and G2. Dashed line represents(closed loop response  $a_1=a_2=\gamma_1=\gamma_2=3$ ),solid line represents(I&I control law**

**$a_1=8,a_2=4,\gamma_1=10,\gamma_2=8$ ),Dotted line represents (open loop response)**

#### 4. CONCLUSION

In this paper we presented a non linear technique immersion and invariance methodology to asymptotically stabilize the SMIB and TMIB systems were modeled using the swing equation model and the CSCs were modelled by the injection model. By observing simulation results we can say that the open loop exhibits heavyand sustained oscillations in  $x_1,x_2$  as represented by dotted lines in figure The closed loop response is plotted for two different sets of tuning parameters. By observing it we can say that the, by increasing the values of 'a' and ' $\gamma$ ' results in peak overshoot and oscillations die out quickly in about 2 seconds. An increase in the values of the tuning parameters results in, decrease in the peak overshoot and the oscillations die out quickly.

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